Circles

NCERT TEXTBOOK QUESTIONS SOLVED

EXERCISE 10.1

Q. 1. *How many tangents can a circle have?*

Sol. A circle can have an infinite number of tangents.

Q. 2. *Fill in the blanks:*

(i) *A tangent to a circle intersects it in .... point(s).*

(ii) *A line intersecting a circle in two points is called a ....*

(iii) *A circle can have .... parallel tangents at the most.*

(iv) *The common point of a tangent to a circle and the circle is called ....*

Sol. (*i*) exactly one (*ii*) secant (*iii*) two (*iv*) point of contact.

Q. 3. *A tangent PQ at a point P of a circle of radius 5 cm meets a line through the centre O at a point Q so that OQ = 12 cm. Length of PQ is:*

(A) 12 cm (B) 13 cm

(C) 8.5 cm (D) $\sqrt{119}$ cm

Sol. Since

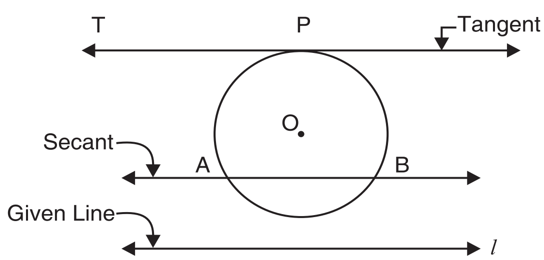
$$PQ = \sqrt{OQ^2 - OP^2}$$
$$= \sqrt{12^2 - 5^2}$$
$$= \sqrt{144 - 25}$$
$$= \sqrt{119}$$

∴ The option (*D*) is correct.

Q. 4. *Draw a circle and two lines parallel to a given line such that one is a tangent and the other a secant to the circle.*

Sol. We have the required figure.

Here, *l* is the given line and a circle with centre *O* is drawn.



The line *PT* is drawn which is parallel to *l* and tangent to the circle.

Also, *AB* is drawn parallel to line *l* and is a secant to the circle.

• Number of Tangents from a Point on a Circle

I. There is **no tangent** to a circle passing through a point lying inside the circle.

II. There is **one and only one tangent** to a circle passing through a point lying on the circle.

III. There are **exactly two tangents** to a circle through a point lying outside the circle.

Theorem 2

The lengths of tangents drawn from an external point to a circle are equal.

[NCERT Exemplar, (CBSE 2010, 2011, 2014, CBSE Foreign 2014)]

Given: We have a circle with centre *O* and a point *P* lying outside the circle. Two tangents *PQ* and *PR* on the circle from *P*.

To Prove: *PR* = *PQ*

Construction: Join *OP*, *OQ* and *OR*

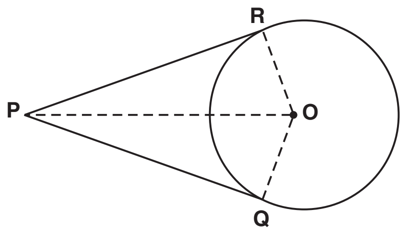
Proof: ∵ *OQ* is a radius and *PQ* is a tangent.

∴ $\angle PQO = 90°$

Similarly, $\angle PRO = 90°$

Now, in right Δ OQP and right Δ ORP, we have:

$$OP = OP \qquad \text{[Common]}$$



$$OQ = OR \qquad \text{[Radii of the same circle]}$$

$$\angle PQO = \angle PRO \qquad \text{[As Proved above]}$$

$\Rightarrow$ $$\Delta OQP \cong \Delta ORP \qquad \text{[R.H.S.]}$$

$\therefore$ Their corresponding parts are equal.

$\Rightarrow$ $$PQ = PR$$

NCERT TEXTBOOK QUESTIONS SOLVED

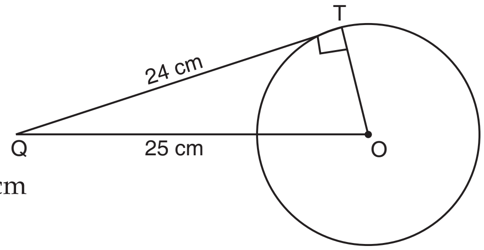
EXERCISE 10.2

Q. 1. *Choose the correct option:*

From a point Q, the length of the tangent to a circle is 24 cm and the distance of Q from the centre is 25 cm. The radius of the circle is

(A) 7 cm *(B) 12 cm*

(C) 15 cm *(D) 24.5 cm*



Sol. $\because$ QT is a tangent to the circle at T and OT is radius

Also, $OQ = 25$ cm and $QT = 24$ cm

$\therefore$ Using Pythagoras theorem, we get

$$OQ^2 = QT^2 + OT^2$$

$$\Rightarrow \quad OT^2 = OQ^2 - QT^2$$

$$= 25^2 - 24^2 = (25 - 24)(25 + 24)$$

$$= 1 \times 49 = 49 = 7^2$$

$$\Rightarrow \quad OT = 7$$

Thus, the required radius is **7 cm.**

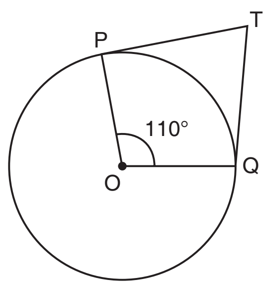
$\therefore$ The correct option is (A).

Q. 2. *Choose the correct option:*

In figure, if TP and TQ are the two tangents to a circle with centre O so that $\angle POQ = 110°$, then $\angle PTQ$ is equal to

(A) 60° *(B) 70°*

(C) 80° *(D) 90°*



Sol. $\because$ TQ and TP are tangents to a circle with centre O.

such that $\angle POQ = 110°$

$\therefore$ $OP \perp PT$ and $OQ \perp QT$

$\Rightarrow \angle OPT = 90°$ and $\angle OQT = 90°$

Now, in the quadrilateral *TPOQ*, we get

$\therefore \quad \angle PTQ + 90° + 110° + 90° = 360°$

$\Rightarrow \quad \angle PTQ + 290° = 360°$

$\Rightarrow \quad \angle PTQ = 360° - 290° = \mathbf{70°}$

Thus, the correct option is (*B*).

Q. 3. *Choose the correct option:*

If tangents PA and PB from a point P to a circle with centre O are inclined to each other at angle of 80°, then ∠POA is equal to

(A) 50° *(B) 60°* *(C) 70°* *(D) 80°*

Sol. Since, *O* is the centre of the circle and two tangents from *P* to the circle are *PA* and *PB*.

$\therefore OA \perp AP$ and $OB \perp BP$

$\Rightarrow \angle OAP = \angle OBP = 90°$

Now, in quadrilateral *PAOB*, we have:

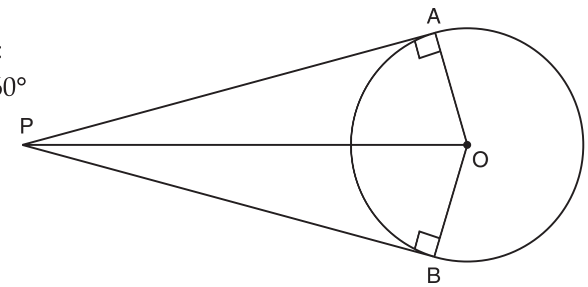
$\angle APB + \angle PAO + \angle AOB + \angle PBO = 360°$

$\Rightarrow 80° + 90° + \angle AOB + 90° = 360°$

$\Rightarrow \quad 260° + \angle AOB = 360°$

$\Rightarrow \quad \angle AOB = 360° - 260°$

$\Rightarrow \quad \angle AOB = 100°.$



In rt Δ *OAP* and rt Δ *OBP*, we have

$OP = OP$ [*Common*]

$\angle OAP = \angle OBP$ [*Each = 90°*]

$OA = OB$ [*Radii of the same circle*]

$\therefore \quad \Delta OAP \cong \Delta OBP$

∴ Their corresponding parts are equal

$\Rightarrow \quad \angle POA = \angle POB$

$\therefore \quad \angle POA = \frac{1}{2}\angle AOB = \frac{1}{2} \times 100° = \mathbf{50°}.$

Thus, the option (*A*) is correct.

Q. 4. *Prove that the tangents drawn at the ends of a diameter of a circle are parallel.*

[CBSE 2012, CBSE Foreign 2014]

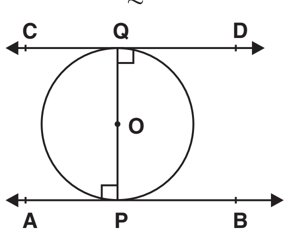
Sol. In the figure, we have:

PQ is diameter of the given circle and *O* is its centre.

Let tangents *AB* and *CD* be drawn at the end points of the diameter *PQ*.

Since the tangent at a point to a circle is perpendicular to the radius through the point.

$\therefore \quad PQ \perp AB \Rightarrow \angle APQ = 90°$



And $\quad PQ \perp CD \Rightarrow \angle PQD = 90°$

$\Rightarrow \quad \angle APQ = \angle PQD$

But they form a pair of alternate angles.

$\therefore \quad AB \parallel CD.$

Q. 5. *Prove that the perpendicular at the point of contact to the tangent to a circle passes through the centre.*

Sol. In the figure, the centre of the circle is O and tangent AB touches the circle at P.

If possible, let PQ be perpendicular to AB such that it is not passing through O.

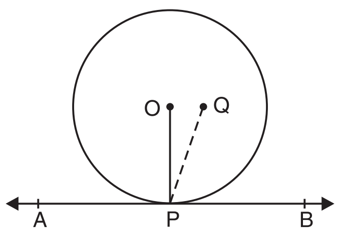
Join OP.

Since tangent at a point to a circle is perpendicular to the radius through that point,

$\therefore \quad AB \perp OP$ i.e. $\angle OPB = 90°$ ...(1)

But by construction,

$AB \perp PQ \Rightarrow \angle QPB = 90°$ ...(2)



From (1) and (2),

$\angle QPB = \angle OPB$

which is possible only when O and Q coincide.

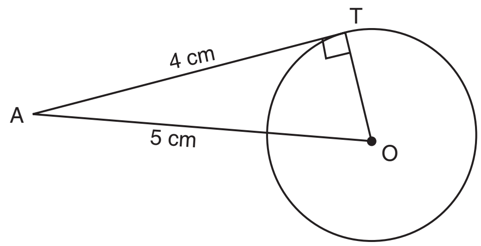
Thus, the perpendicular at the point of contact to the tangent passes through the centre.

Q. 6. *The length of a tangent from a point A at distance 5 cm from the centre of the circle is 4 cm. Find the radius of the circle.*

Sol. $\because$ The tangent to a circle is perpendicular to the radius through the point of contact.

$\therefore \quad \angle OTA = 90°$

Now, in the right $\Delta\, OTA$, we have:



$OP^2 = OT^2 + PT^2$

$\Rightarrow \quad 5^2 = OT^2 + 4^2$

$\Rightarrow \quad OT^2 = 5^2 - 4^2$

$\Rightarrow \quad OT^2 = (5 - 4)\,(5 + 4)$

$\Rightarrow \quad OT^2 = 1 \times 9 = 9 = 3^2$

$\Rightarrow \quad OT = 3$

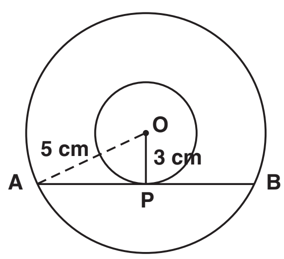
Thus, the radius of the circle is **3 cm.**

Q. 7. *Two concentric circles are of radii 5 cm and 3 cm. Find the length of the chord of the larger circle which touches the smaller circle.*

Sol. In the figure, O is the common centre, of the given concentric circles.

AB is a chord of the bigger circle such that it is a tangent to the smaller circle at P.

Since OP is the radius of the smaller circle through P, the point of contact,



$\therefore \quad OP \perp AB$

$\Rightarrow \quad \angle APB = 90°$

Also, a radius perpendicular to a chord bisects the chord.

$\therefore$ OP bisects $AB \Rightarrow AP = \frac{1}{2}AB$

Now, in right Δ APO,

$$OA^2 = AP^2 - OP^2$$

$$\Rightarrow \quad 5^2 = AP^2 - 3^2$$

$$\Rightarrow \quad AP^2 = 5^2 - 3^2$$

$$\Rightarrow \quad AP^2 = (5 - 3)(5 + 3) = 2 \times 8$$

$$\Rightarrow \quad AP^2 = 16 = (4)^2$$

$$\Rightarrow \quad AP = 4 \text{ cm}$$

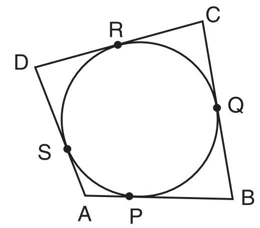
$$\Rightarrow \quad \frac{1}{2}AB = 4 \quad \Rightarrow \quad AB = 2 \times 4 = 8 \text{ cm}$$

Hence, the required length of the chord AB is **8 cm.**

Q. 8. *A quadrilateral ABCD is drawn to circumscribe a circle (see figure). Prove that:*

$$AB + CD = AD + BC$$ *[CBSE (Foreign) 2014, CBSE 2012] (AI CBSE 2008 C)*

Sol. Since the sides of quadrilateral $ABCD$, *i.e.*, AB, BC, CD and DA touch the circle at P, Q, R and S respectively, and the lengths of two tangents to a circle from an external point are equal.



$\therefore$
$$AP = AS$$
$$BP = BQ$$
$$DR = DS$$
$$CR = CQ$$

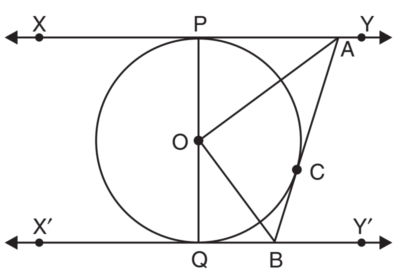
Adding them, we get

$(AP + BP) + (CR + RD) = (BQ + QC) + (DS + SA)$

$$\Rightarrow \quad AB + CD = BC + DA$$

which was to be proved.

Q. 9. *In the figure, XY and X′Y′ are two parallel tangents to a circle with centre O and another tangent AB with point of contact C intersecting XY at A and X′Y′ at B. Prove that $\angle AOB = 90°$.*



[CBSE 2012]

Sol. $\because$ The tangents drawn to a circle from an external point are equal.

$\therefore$ $$AP = AC$$

In Δ PAO and Δ AOC, we have:

$AO = AO$ [*Common*]

$OP = OC$ [*Radii of the same circle*]

$AP = AC$

$\Rightarrow \quad \Delta PAO \cong \Delta AOC$ [SSS Congruency]

$\therefore \quad \angle PAO = \angle CAO$

$\angle PAC = 2\angle CAO$ ...(1)

Similarly $\quad \angle CBQ = 2\angle CBO$ ...(2)

Again, we know that sum of internal angles on the same side of a transversal is 180°.

$\therefore \quad \angle PAC + \angle CBQ = 180°$

$\Rightarrow \quad 2\angle CAO + 2\angle CBO = 180°$ [From (1) and (2)]

$\Rightarrow \quad \angle CAO + \angle CBO = \frac{180°}{2} = 90°$ ...(3)

Also $\quad \angle CAO + \angle CBO + \angle AOB = 180°$ [Sum of angles of a triangle]

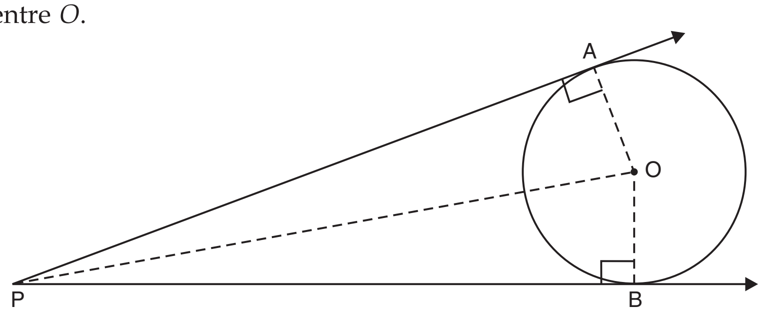
$\Rightarrow \quad 90° + \angle AOB = 180°$

$\Rightarrow \quad \angle AOB = 180° - 90°$

$\Rightarrow \quad \angle AOB = 90°.$

Q. 10. *Prove that the angle between the two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line-segment joining the points of contact at the centre.*

Sol. Here, let *PA* and *PB* be two tangents drawn from an external point *P* to a circle with centre *O*.



Now, in right ΔOAP and right ΔOBP, we have

$PA = PB$ [Tangents to circle from an external point P]

$OA = OB$ [Radii of the same circle]

$OP = OP$ [Common]

$\therefore$ By *SSS* congruency,

$\Delta OAP \cong \Delta OBP$

$\therefore$ Their corresponding parts are equal.

$\therefore \quad \angle OAA = \angle OPB$

And $\quad \angle AOP = \angle BOP$

$\Rightarrow \quad \angle APB = 2\angle OPA$ and $\angle AOB = 2\angle AOP$

But $\quad \angle AOP = 90° - \angle OPA$

$\Rightarrow \quad 2\angle AOP = 180° - 2\angle OPA$

$\Rightarrow \quad \angle AOB = 180° - \angle APB$

$\Rightarrow$ $\boldsymbol{\angle AOB + \angle APB = 180°.}$

Q. 11. *Prove that the parallelogram circumscribing a circle is a rhombus.* (CBSE 2012, CBSE Delhi 2014)

Sol. We have *ABCD*, a parallelogram which circumscribes a circle (*i.e.,* its sides touch the circle) with centre *O*.

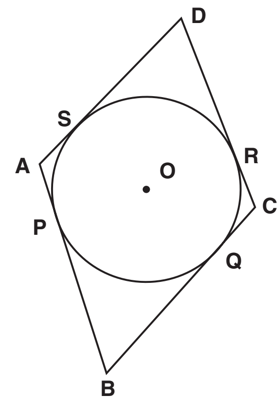
Since tangents to a circle from an external point are equal in length,

$\therefore \quad AP = AS$

$BP = BQ$

$CR = CQ$

$DR = DS$



Adding, we get

$(AP + BP) + (CR + DR) = (AS + DS) + (BQ + CQ)$

$\Rightarrow \quad AB + CD = AD + BC$

But $\quad AB = CD \quad$ [opposite sides of *ABCD*]

and $\quad BC = AD$

$\therefore \quad AB + CD = AD + BC \quad \Rightarrow \quad 2\,AB = 2\,BC$

$\Rightarrow \quad AB = BC$

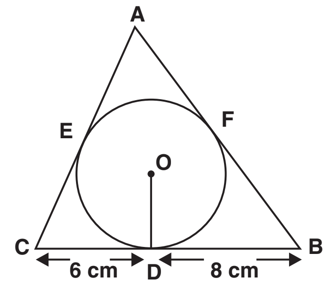
Similarly $\quad AB = DA$ and $DA = CD$

Thus, $\quad AB = BC = CD = AD$

Hence ***ABCD* is a rhombus.**

Q. 12. *A triangle ABC is drawn to circumscribe a circle of radius 4 cm such that the segments BD and DC into which BC is divided by the point of contact D are of lengths 8 cm and 6 cm respectively (see figure). Find the sides AB and AC.*

[CBSE 2012]



Sol. Here $\Delta\, ABC$ subscribe the circle with centre *O*.

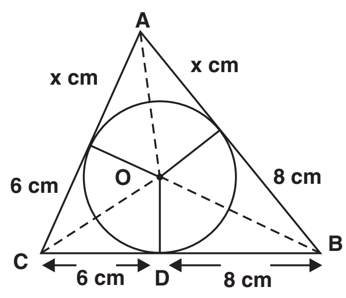
Also, radius = 4 cm

$\because$ The sides *BC*, *CA* and *AB* touch the circle at *D*, *E* and *F* respectively.

$\therefore \quad BF = BD = 8$ cm

$CE = CD = 6$ cm

$AF = AE = x$ cm (say)



$\Rightarrow$ The sides of the triangle are:

14 cm, $(x + 6)$ cm and $(x + 8)$ cm

Perimeter of $\Delta\, ABC$

$= [14 + (x + 6) + (x + 8)]$ cm

$= [14 + 6 + 8 + 2x]$ cm

$= 28 + 2x$ cm

$\Rightarrow$ Semi perimeter of $\Delta\, ABC$

$$S = \frac{1}{2}\,[28 + 2x] \text{ cm} = (14 + x) \text{ cm}$$

$\therefore \quad S - AB = (14 + x) - (8 + x) = 6$

$S - BC = (14 + x) - (14) = x$

$S - AC = (14 + x) - (16 + x) = 8$

$$\therefore \quad \text{Area of } \Delta\, ABC = \sqrt{S\,(S - AB)\,(S - BC)\,(S - AC)} = \sqrt{(14 + x)\,(6)\,(x)\,(8)} \text{ cm}^2$$

$$= \sqrt{(14+x)\,48x} \text{ cm}^2 \qquad \text{...(1)}$$

Now, $\text{ar}(\Delta OBC) = \frac{1}{2} \times BC \times OD$

$$= \frac{1}{2} \times 14 \times 4 \qquad [\because OD = Radius]$$

$$= 28 \text{ cm}^2$$

$$\text{ar}(\Delta OCA) = \frac{1}{2} CA \times OE = \frac{1}{2} \times (x+6) \times 4$$

$$= \frac{1}{2} \times 4(x+6) = (2x+12) \text{ cm}^2$$

$$\text{ar}(\Delta OAB) = \frac{1}{2} \times AB \times OF = \frac{1}{2} \times (x+8) \times 4$$

$$= (2x+16) \text{ cm}^2$$

$\therefore$ $\text{ar}(\Delta ABC) = \text{ar}(\Delta OBC) + \text{ar}(\Delta OCA) + \text{ar}(\Delta OAB)$

$$= 28 \text{ cm}^2 + (2x+12) \text{ cm}^2 + (2x+16) \text{ cm}^2$$

$$= (28+12+16) + 4x \text{ cm}^2$$

$$= (56+4x) \text{ cm}^2 \qquad \text{...(2)}$$

From (1) and (2), we have:

$$56 + 4x = \sqrt{(14+x)\,48x}$$

$$4[14+x] = 4\sqrt{(14+x) \times 3x}$$

$$\Rightarrow 14 + x = \sqrt{(14+x)\,3x}$$

Squaring both sides

$$(14+x)^2 = (14+x)\,3x$$

$$\Rightarrow 196 + x^2 + 28x = 42x + 3x^2$$

$$\Rightarrow 2x^2 + 14x - 196 = 0 \Rightarrow x^2 + 7x - 98 = 0$$

$$\Rightarrow (x-7)(x+14) = 0$$

$\Rightarrow$ Either $x - 7 = 0 \Rightarrow x = 7$

or $x + 14 = 0 \Rightarrow x = (-14)$

But $x = (-14)$ is not required

$\therefore$ $x = 7$ cm

Thus, $AB = 8 + 7 = 15$ cm

$BC = 8 + 6 = 14$ cm

$CA = 6 + 7 = 13$ cm.

Q. 13. *Prove that opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.* *[CBSE 2012]*

Sol. We have a circle with centre *O*.

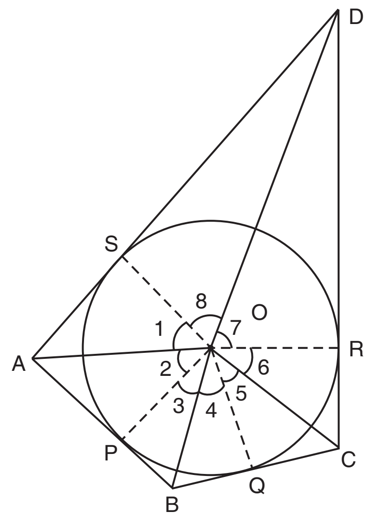
A quadrilateral *ABCD* is such that the sides *AB*, *BC*, *CD* and *DA* touch the circle at *P*, *Q*, *R* and *S* respectively.

Let us join *OP*, *OQ*, *OR* and *OS*. We know that two tangents drawn from an external point to a circle subtend equal angles at the centre.

$\therefore \quad \angle 1 = \angle 2$

$\angle 3 = \angle 4$

$\angle 5 = \angle 6$ and $\angle 7 = \angle 8$



Also, the sum of all the angles around a point is 360°.

$\therefore \angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 + \angle 7 + \angle 8 = 360°$

$\therefore \quad 2[\angle 1 + \angle 8 + \angle 5 + \angle 4] = 360°$

$\Rightarrow \quad (\angle 1 + \angle 8 + \angle 5 + \angle 4) = 180° \quad (1)$

And $\quad 2[\angle 2 + \angle 3 + \angle 6 + \angle 7] = 360°$

$\Rightarrow \quad (\angle 2 + \angle 3) + (\angle 6 + \angle 7) = 180° \quad (2)$

Since, $\angle 2 + \angle 3 = \angle AOB$

$\angle 6 + \angle 7 = \angle COD$

$\angle 1 + \angle 8 = \angle AOD$

$\angle 4 + \angle 5 = \angle BOC$

$\therefore$ From (1) and (2), we have:

$\angle AOD + \angle BOC = 180°$ and

$\angle AOB + \angle COD = 180°$

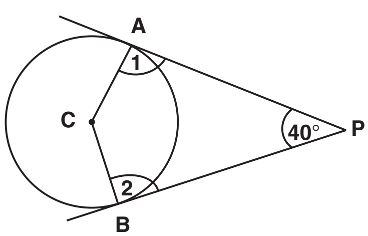
MORE QUESTIONS SOLVED

I. VERY SHORT ANSWER TYPE QUESTIONS

Q. 1. *In the adjoining figure, PA and PB are tangents from P to a circle with centre C. If $\angle APB = 40°$ then find $\angle ACB$.*

Sol. Since a tangent to a circle is perpendicular to the radius through the point of contact,

$\therefore \quad \angle 1 = 90°$ and $\angle 2 = 90°$



Now, in quadrilateral *APBC*, we have:

$\angle 1 + \angle ACB + \angle 2 + \angle P = 360°$

$\Rightarrow \quad 90° + \angle ACB + 90° + 40° = 360°$

$\Rightarrow \quad \angle ACB + 220° = 360°$

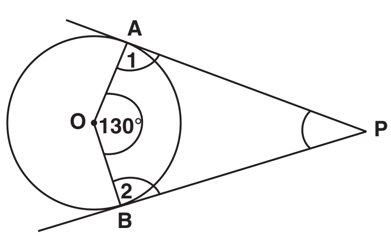
$\Rightarrow \quad \angle ACB = 360° - 220°$

$= \mathbf{140°}$.

Q. 2. *In the given figure, PA and PB are tangents from P to a circle with centre O. If $\angle AOB = 130°$, then find $\angle APB$.*

Sol. Since a tangent to a circle is perpendicular to the radius through the point of contact,

$\therefore \quad \angle 1 = \angle 2 = 90°$



Now, in quadrilateral *AOBP*, we have:

$\angle 1 + \angle AOB + \angle 2 + \angle APB = 360°$

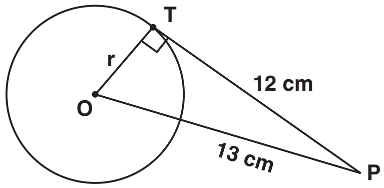
$\Rightarrow \quad 90° + 130° + 90° + \angle APB = 360°$

$\Rightarrow \quad 310° + \angle APB = 360°$

$\Rightarrow \quad \angle APB = 360 - 310 = 50°$

Thus, $\angle APB = 50°$.

Q. 3. *In the given figure, PT is a tangent to a circle whose centre is O. If PT = 12 cm and PO = 13 cm then find the radius of the circle.*



Sol. Since a tangent to a circle is perpendicular to the radius through the point of contact,

$\therefore \quad \angle OTP = 90°$

In rt Δ OTP, using Pythagoras theorem, we get

$$OP^2 = OT^2 + PT^2$$

$$\Rightarrow \quad 13^2 = OT^2 + 12^2$$

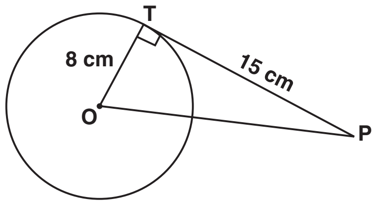
$$\Rightarrow \quad OT^2 = 13^2 - 12^2 = (13 - 12)(13 + 12) = 1 \times 25 = 25$$

$$\therefore \quad OT^2 = 5^2$$

$$\Rightarrow \quad OT = 5$$

Thus, radius (r) = **5 cm.**

Q. 4. *In the given figure, PT is a tangent to the circle and O is its centre. Find OP.*



Sol. Since, a tangent to a circle is perpendicular to the radius through the point of contact.

$\therefore \quad \angle OTP = 90°$

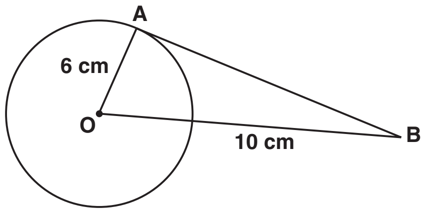
In right Δ OTP, using Pythagoras theorem, we get

$$OP^2 = OT^2 + PT^2$$

$$= 8^2 + 15^2 = 64 + 225 = 289 = 17^2$$

$$\Rightarrow \quad OP = \sqrt{17^2} = \mathbf{17 \text{ cm.}}$$

Q. 5. *If O is the centre of the circle, then find the length of the tangent AB in the given figure.*



Sol. $\because$ A tangent to a circle is perpendicular to the radius through the point of contact.

$\therefore \quad \angle OAB = 90°$

Now, in right Δ OAB, we have

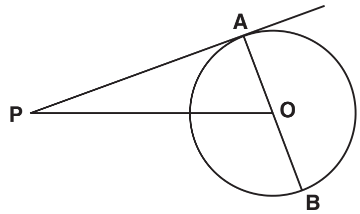
$$OB^2 = OA^2 + AB^2$$

$$\Rightarrow \quad 10^2 = 6^2 + AB^2$$

$$\Rightarrow \quad AB^2 = 10^2 - 6^2 = (10 - 6)(10 + 6) = 4 \times 16 = 64 = 8^2$$

$$\Rightarrow \quad AB = \sqrt{8^2} = \mathbf{8}.$$

Q. 6. *In the figure, PA is a tangent from an external point P to a circle with centre O. If ∠POB = 115° then find ∠APO.* (AI CBSE 2009 C)



Sol. Here, PA is a tangent and OA is radius. Also, a radius through the point of contact is perpendicular to the tangent.

$\therefore \quad OA \perp PA$

$\Rightarrow \quad \angle PAO = 90°$

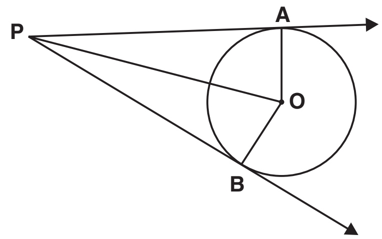
In ΔOAP, $\angle POB$ is an external angle,

$\therefore \quad \angle APO + \angle PAO = \angle POB$

$\Rightarrow \quad \angle APO + 90° = 115°$

$\Rightarrow \quad \angle APO = 115° - 90° = \mathbf{25°}$

Q. 7. *In the following figure, PA and PB are tangents drawn from a point P to the circle with centre O. If ∠APB = 60°, then what is ∠AOB?* (CBSE 2009 C)



Sol. The radius of the circle through the point of contact is perpendicular to the tangent.

$\therefore OA \perp AP$ and $OB \perp BP$

$\Rightarrow \angle PAO = \angle PBO = 90°$

Now, in quadrilateral $OAPB$,

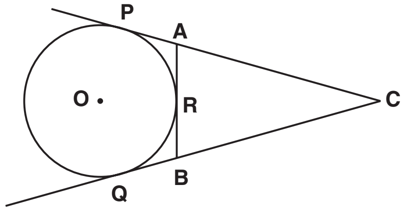
$$\angle OAP + \angle APB + \angle PBO + \angle AOB = 360°$$

$$90° + 60° + 90° + \angle AOB = 360°$$

$\Rightarrow \quad \angle AOB + 240° = 360°$

$\Rightarrow \quad \angle AOB = 360° - 240° = 120°$

Q. 8. *In the figure, CP and CQ are tangents to a circle with centre O. ARB is another tangent touching the circle at R. If QC = 11 cm, BC = 7 cm then find, the length of BR.* (CBSE 2009)



Sol. ∵ Tangents drawn from an external point are equal,

∴ $BQ = BR$ ...(1)

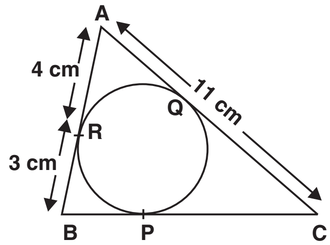
And $CQ = CP$

Since, $BC + BQ = QC$

⇒ $7 + BR = 11$ [∵ $BQ = BR$]

$BR = 11 - 7 = $ **4 cm.**

Q. 9. *In the figure, ΔABC is circumscribing a circle. Find the length of BC.* (AI CBSE 2009)



Sol. Since tangents drawn from an external point to the circle are equal,

∴ $AR = AQ = 4$ cm ...(1)

$BR = BP = 3$ cm ...(2)

$PC = QC$ ...(3)

∴ $QC = AC - AQ$

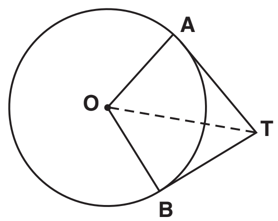
$= 11 - 4 = 7$ cm [From (1)]

$BC = BP + PC$ [From (3)]

$= 3 + QC$

$= (3 + 7)$ cm $=$ **10 cm**

Q. 10. *In the figure, if ∠ATO = 40°, find ∠AOB.* [AI CBSE 2008]



Sol. Since the tangent is perpendicular to the radius through the point of contact,

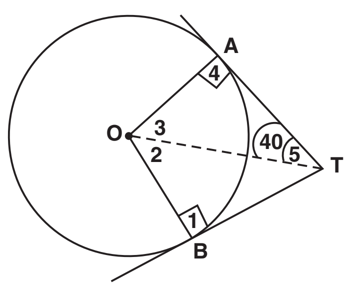
∴ $\angle 1 = \angle 4 = 90°$

Also, $OA = OB$ [*Radii of the same circle*]

$OT = OT$ [*Common*]

∴ $\Delta OAT \cong \Delta OBT$ [*RHS congruency*]

⇒ $\angle 3 = \angle 2$



Now, in ΔOAT,

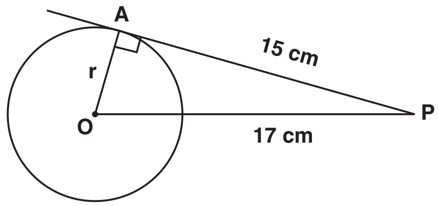
$$\angle 3 + \angle 4 + \angle 5 = 180°$$

$$\Rightarrow \quad \angle 3 + 90° + 40° = 180°$$

$$\Rightarrow \quad \angle 3 = 180° - 90° - 40° = 50°$$

$$\Rightarrow \quad \angle AOB = 50° + 50° = \mathbf{100°}.$$

Q. 11. *From a point P, the length of the tangent to a circle is 15 cm and distance of P from the centre of the circle is 17 cm, then what is the radius of the circle?* [CBSE 2008 C]



Sol. Since radius is perpendicular to the tangent through the point of contact,

$$\therefore \quad OA \perp AP$$

$$\Rightarrow \quad \angle OAP = 90°$$

In rt ΔOAP, we have:

$$OA^2 + AP^2 = OP^2$$

$$\Rightarrow \quad r^2 + (15)^2 = (17)^2$$

$$r^2 = 17^2 - 15^2 = (17 - 15)(17 + 15) = 2 \times 32 = 64$$

$$\Rightarrow \quad r = \sqrt{64} = 8$$

Thus, radius = 8 cm.

Q. 12. *The two tangents from an external point P to a circle with centre O are PA and PB. If $\angle APB = 70°$, then what is the value of $\angle AOB$?* (AI CBSE 2008 C)

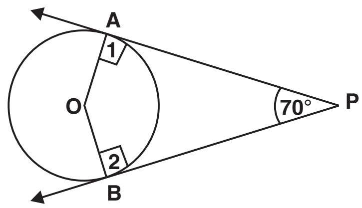
Sol. Since tangent is perpendicular to the radius through the point of contact.

$$\therefore \quad \angle 1 = \angle 2 = 90°$$

In quadrilateral $OABP$,

$$\angle AOB + \angle 1 + \angle 2 + \angle APB = 360°$$

$$\angle AOB + 90° + 90° + 70° = 360°$$



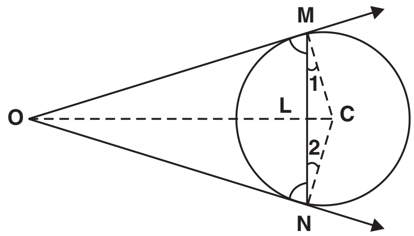
$$\Rightarrow \quad \angle AOB + 250° = 360°$$

$$\Rightarrow \quad \angle AOB = 360° - 250° = \mathbf{110°}$$

II. SHORT ANSWER TYPE QUESTIONS

Q. 1. *Prove that the tangents drawn at the ends of a chord of a circle make equal angles with the chord.* *[NCERT Exemplar]*

Sol.



Let NM be a chord of a circle with centre C.

Let the tangents at M and N meet at O.

$\because$ OM is a tangent at M

$\therefore$ $\angle OMC = 90°$ ...(1)

Similarly $\angle ONC = 90°$ ...(2)

Since, $CM = CN$ [Radii of the same circle]

$\therefore$ In Δ CMN, $\angle 1 = \angle 2$

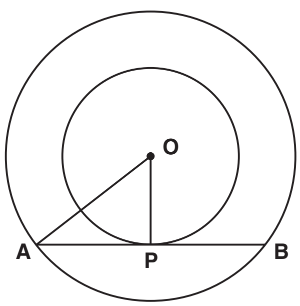
From (1) and (2), we have

$$\angle OMC - \angle 1 = \angle ONC - \angle 2$$

$$\Rightarrow \angle OML = \angle ONL$$

Thus, tangents make equal angles with the chord.

Q. 2. *Two concentric circles have a common centre O. The chord AB to the bigger circle touches the smaller circle at P. If OP = 3 cm and AB = 8 cm then find the radius of the bigger circle.*



Sol. $\because$ *AB* touches the smaller circle at *P*.

$\therefore$ $OP \perp AB \Rightarrow \angle OPA = 90°$

Now, *AB* is a chord of the bigger circle.

Since, the perpendicular from the centre to a chord, bisects the chord,

$\therefore$ *P* is the mid-point of *AB*

$$\Rightarrow AP = \frac{8}{2} = 4 \text{ cm}$$

In right Δ *APO*, we have

$$AO^2 = OP^2 + AP^2$$

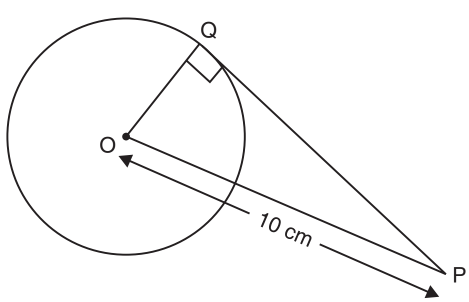
$$\Rightarrow AO^2 = 3^2 + 4^2$$

$$\Rightarrow AO^2 = 9 + 16 = 25 = 5^2$$

$$\Rightarrow AO = \sqrt{5^2} = 5 \text{ cm}$$

Thus, the radius of the bigger circle is **5 cm.**

Q. 3. *In the given figure, O is the centre of the circle and PQ is a tangent to it. If its circumference is 12π cm, then find the length of the tangent.*



Sol. $\because$ Circumference of the circle = 12π cm

$\therefore$ $2\pi r = 12\pi$

[$\because$ *r is the radius of the circle*]

$$\Rightarrow \quad r = \frac{12\pi}{2\pi} = 6 \text{ cm}$$

$\Rightarrow$ Radius of the circle = 6 cm = OQ

Since a tangent to circle is perpendicular to the radius through the point of contact,

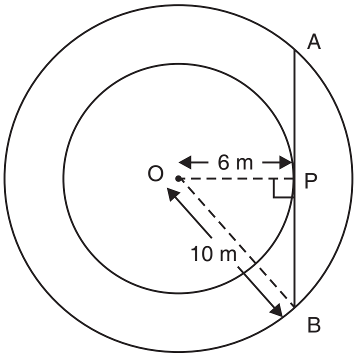
$\therefore \quad \angle OQP = 90°$

Now, in rt Δ OQP, we have:

$$OQ^2 + QP^2 = OP^2$$
$$\Rightarrow \quad 6^2 + QP^2 = 10^2$$
$$\Rightarrow \quad QP^2 = 10^2 - 6^2 = (10 - 6)(10 + 6) = 4 \times 16 = 64 = 8^2$$
$$\Rightarrow \quad QP = \sqrt{8^2} = 8$$

Thus, the length of the tangent is **8 cm.**

Q. 4. *Given two concentric circles of radii 10 cm and 6 cm. Find the length of the chord of the larger circle which touches the other circle.*



Sol. The chord AB touches the inner circle at P.

$\therefore$ AB is tangent to the inner circle.

$\Rightarrow \quad OP \perp AB$

[$\because$ *O is the centre and OP is radius through the point of contact P*]

$\therefore \quad \angle OPB = 90°$.

Now, in right Δ OPB, we have:

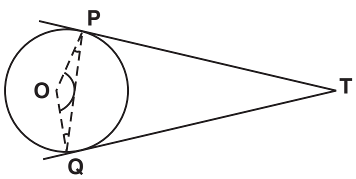
$$OP^2 + PB^2 = OB^2$$
$$\Rightarrow \quad 6^2 + PB^2 = 10^2$$
$$\Rightarrow \quad PB^2 = 10^2 - 6^2 = (10 - 6) \times (10 + 6)$$
$$\Rightarrow \quad PB^2 = 4 \times 16$$
$$\Rightarrow \quad PB^2 = 64 = 8^2$$
$$\Rightarrow \quad PB = \sqrt{8^2} = 8 \text{ cm}$$

$\because$ The radius perpendicular to a chord bisects the chord.

$\therefore$ P is the mid-point of AB

$\therefore$ $AB = 2 \times PB = 2 \times 8 =$ **16 cm.**

Q. 5. *Two tangents TP and TQ are drawn to a circle with centre O from an external point T. Prove that $\angle PTQ = 2\ \angle OPQ$.* (CBSE Sample Paper 2011)



Sol. $\because$ Tangent to a circle is perpendicular to the radius through the point of contact.

In quadrilateral. $OPTQ$,

$\angle OPT + \angle OQT + \angle POQ + \angle PTQ = 360°$

or $90° + 90° + \angle POQ + \angle PTQ = 360°$

$\Rightarrow \angle POQ + \angle PTQ = 360° - 90° - 90° = 180°$ ...(1)

In Δ OPQ, $\angle 1 + \angle 2 + \angle POQ = 180°$ ...(2)

Since $OP = OQ$ [*Radii of the same circle*]

$\Rightarrow$ $\angle 1 = \angle 2$ [*Angles opposite to equal sides*]

$\therefore$ $\angle OPT = 90° = \angle OQT$

$\therefore$ From (2), we have

$$\angle 1 + \angle 1 + \angle POQ = 180°$$

$$\Rightarrow \quad 2\angle 1 + \angle POQ = 180° \quad ...(3)$$

From (1) and (3), we have

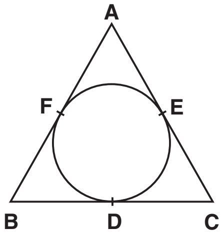
$$2\angle 1 + \angle POQ = \angle POQ + \angle PTQ$$

$$\Rightarrow \quad 2\angle 1 = \angle PTQ$$

$$\Rightarrow \quad \mathbf{2\angle OPQ = \angle PTQ.}$$

Q. 6. *In the figure, the incircle of Δ ABC touches the sides BC, CA and AB at D, E and F respectively. If AB = AC, prove that BD = CD.*

Sol. Since the lengths of tangents drawn from an external point to a circle are equal,



$\therefore$ We have $AF = AE$

$BF = BD$

$CD = CE$

Adding them, we get

$(AF + BF) + CD = (AE + CE) + BD$

$\Rightarrow$ $AB + CD = AC + BD$

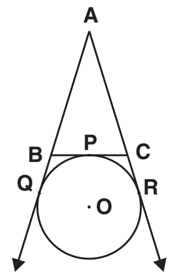
But $AB = AC$ (Given)

$\therefore$ $CD = BD$.

Q. 7. *A circle is touching the side BC of a Δ ABC at P and touching AB and AC produced at Q and R. Prove that:*

$AQ = \frac{1}{2}$ *(Perimeter of Δ ABC)* *[NCERT Exemplar CBSE 2011, 2012]*

Sol. Since, the two tangents drawn to a circle from an external point are equal.



$\therefore$ AQ = AR ...(1)

Similarly,

BQ = BP ...(2)

and CR = CP ...(3)

Now, Perimeter of Δ ABC

= AB + BC + AC

= AB + (BP + PC) + AC

= AB + (BQ + CR) + AC [From (2) and (3)]

= (AB + BQ) + (CR + AC)

= AQ + AR

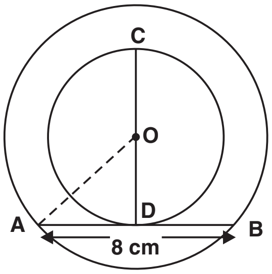
= AQ + AQ [From (1)]

= 2AQ

$\Rightarrow$ $AQ = \frac{1}{2}$ (Perimeter Δ ABC)

Q. 8. *In two concentric circles, a chord of the larger circle touches the smaller circle. If the length of this chord is 8 cm and the diameter of the smaller circle is 6 cm, then find the diameter of the larger circle.* (CBSE 2009 C)

Sol. Let the common centre be O. Let AB be the chord of the larger circle.



$\therefore \quad AB = 8$ cm

And CD is the diameter of the smaller circle *i.e.,*

$CD = 6$ cm

$\Rightarrow \quad OD = \frac{1}{2}(6) = 3$ cm

Join OA. D is the point of contact.

$\therefore \quad OD \perp AB$

$\Rightarrow D$ is the mid point of AB

$\Rightarrow \quad AD = 4$ cm

Now, in right ΔADO, we have:

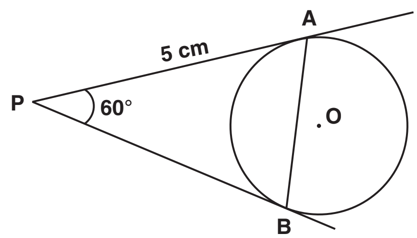
$$AO^2 = AD^2 + OD^2$$
$$= 4^2 + 3^2 = 16 + 9 = 25 = 5^2$$

$\Rightarrow \quad AO = 5$ cm

$\Rightarrow \quad 2AO = 2(5 \text{ cm}) = 10$ cm

$\therefore$ The diameter of the bigger circle is **10 cm.**

Q. 9. *In the following figure, PA and PB are two tangents drawn to a circle with centre O, from an external point P such that PA = 5 cm and ∠APB = 60°. Find the length of chord AB.* (CBSE 2009 C)



Sol. Since the tangents to a circle from an external point are equal,

$\therefore \quad PA = PB = 5$ cm

In ΔPAB, we have

$\angle PAB = \angle PBA \quad [\because PA = PB]$

$\therefore \quad \angle PAB + \angle PBA + \angle APB = 180°$

$\Rightarrow \quad \angle PAB + \angle PAB + 60° = 180°$

$\Rightarrow \quad 2\angle PAB + 60° = 180°$

$\Rightarrow \quad 2\angle PAB = 180° - 60° = 120°$

$\Rightarrow \quad \angle PAB = 60°$

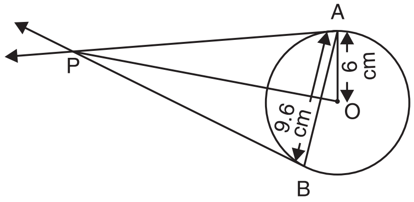
$\Rightarrow$ Each angle of ΔPAB is 60°.

$\Rightarrow$ ΔPAB is an equilateral triangle.

$\therefore \quad PA = PB = AB = 5$ cm

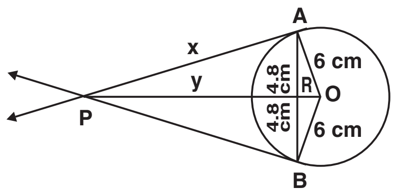
Thus, $\quad AB =$ **5 cm**

Q. 10. *In the following figure, AB is a chord of length 9.6 cm of a circle with centre O and radius 6 cm.*



The tangents at A and B intersect at P. Find the length PA. (AI CBSE 2009 C)

Sol.



Join OB.

Let $PA = x$ cm and $PR = y$ cm

Since, OP is perpendicular bisector of AB

$\therefore \quad AR = BR = \dfrac{9.6}{2} = 4.8$ cm

Now, in rt ΔOAR, we have:

$OA^2 = OR^2 + AR^2$ [*By Pythagoras theorem*]

$\Rightarrow \quad OR^2 = OA^2 - AR^2$

$= 6^2 - (4.8)^2 = (6 - 4.8) \times (6 + 4{\cdot}8) = 1.2 \times 10.8$

$\Rightarrow \quad = 12.96$

$OR = 3.6$ cm.

Again, in right ΔOAP,

$OP^2 = AP^2 + OA^2$

$OP^2 = (AR^2 + PR^2) + OA^2$ [$\because AP^2 = AR^2 + PR^2$]

$\Rightarrow \quad (y + 3.6)^2 = (4.8)^2 + y^2 + 6^2$

$\Rightarrow \quad y^2 + 12.96 + 7.2\,y = 23.04 + y^2 + 36$

$\Rightarrow \quad 7.2\,y = 46.08$

$\Rightarrow \quad y = \dfrac{46.08}{7.2} = 6.4$

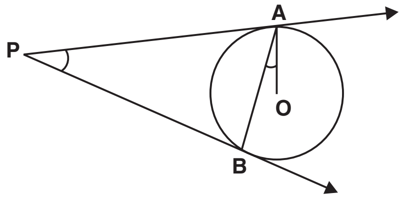
$\Rightarrow \quad PR = 6.4$ cm

Now, $\quad AP^2 = AP^2 + PR^2$

$= (4.8)^2 + (6.4)^2 = 23.04 + 40.96 = 64$

$\Rightarrow \quad AP = \sqrt{64}$ **= 8 cm**

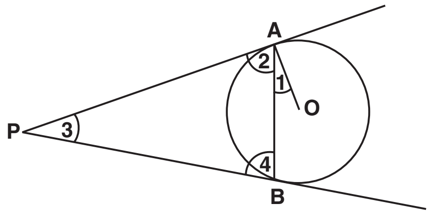
Q. 11. *Two tangents PA and PB are drawn to a circle with centre O from an external point P. Prove that*
$$\angle APB = 2\angle OAB$$
(CBSE 2009)



Sol. We have PA and PB, the tangents to the circle and O is the centre of the circle.

$\therefore \quad PA = PB$

$\Rightarrow \quad \angle 2 = \angle 4$ ...(1)



Since the tangent is perpendicular to the radius through the point of contact,

$\therefore \quad \angle OAP = 90°$

$\Rightarrow \quad \angle 1 + \angle 2 = 90°$ ...(2)

$\Rightarrow \quad \angle 2 = 90° - \angle 1$

Now, in ΔABP, we have:

$\therefore \quad \angle 2 + \angle 3 + \angle 4 = 180°$

$\Rightarrow \quad \angle 2 + \angle 3 + \angle 2 = 180°$ [*From (1)*]

$\Rightarrow \quad \angle 2 + \angle 3 = 180°$

$\Rightarrow \quad 2\ (90° - \angle 1) + \angle 3 = 180°$ [*From (2)*]

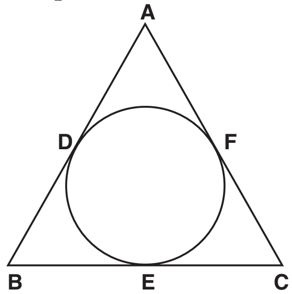
$\Rightarrow \quad 180° - 2\ \angle 1 + \angle 3 = 180°$

$\Rightarrow \quad 2\ \angle 1 = \angle 3 \quad \Rightarrow \quad \angle 3 = 2\angle 1$

$\Rightarrow \quad \angle APB = 2\angle OAB$

Q. 12. *ABC is an isosceles triangle, in which AB = AC, circumscribed about a circle. Show that BC is bisected at the point of contact.* *[CBSE 2012]*

Sol. We know that the tangents to a circle from an external point are equal.



$\therefore \quad AD = AF$

Similarly,

$BD = BE$

and $\quad CE = CF$

Since $\quad AB = AC$ [*Given*]

$\Rightarrow \quad AB - AD = AC - AD$

$\Rightarrow \quad AB - AD = AC - AF$ $[\because AD = AF]$

$\Rightarrow$ $BD = CF$ ...(1)

But $BF = BD$ and $CF = CE$

$\therefore$ From (1), we have:

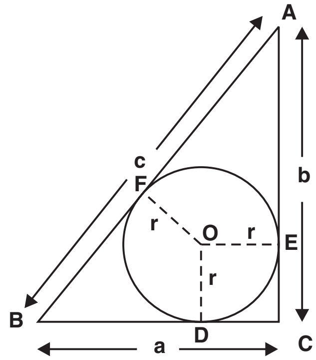
$$BE = CE$$

Q. 13. *If a, b, c are the sides of a right triangle where c is hypotenuse, prove that the radius r of the circle which touches the sides of the triangle is given by* $r = \frac{a+b-c}{2}$ [(NCERT Exemplar) (CBSE 2012)]

Sol. Here, a, b and c are the sides of rt Δ ABC

Such that BC = a, CA = b and AB = c

Let the circle touches the sides BC, CA, AB at D, E and F respectively.



= AE = AF and BD = BF

Also, CE = CD = r

$\therefore$ AF = $b - r$

BF = $a - r$

Now, AB = c $\Rightarrow$ (AF + BF) = $(b - r) + (a - r)$

$\Rightarrow$ $c = b + a - 2r$

$\Rightarrow$ $2r = a + b - c$

$\Rightarrow$ $r = \frac{a+b-c}{2}$

Q. 14. *In a right ΔABC, right angled at B, BC = 5 cm and AB = 12 cm. The circle is touching the sides of Δ ABC. Find the radius of the circle.* [CBSE 2014]

Sol. Let the circle with centre O and radius 'r' touches AB, BC and AC at P, Q, R, respectively.

Now,

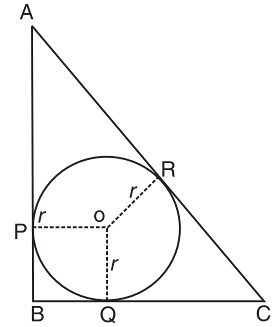
AR = AP

$\because$ AP = AB – BP = $(12 - r)$ cm

$\therefore$ AR = $(12 - r)$cm

Similarly, CR = $(5 - r)$cm

Now, using Pythagoras theorem in rt Δ ABC, we have



$$AC^2 = AB^2 + BC^2$$

$\Rightarrow$ $AC^2 = 12^2 + 5^2$

$\Rightarrow$ AC = 13 cm

But AC = AR + CR

= (12 – r) + (5 – r)

$\Rightarrow$ (12 – r) + (5 – r) = 13 cm

$\Rightarrow$ 17 – 2r = 13 cm

$\Rightarrow$ 2 r = 17 – 13 = 4 cm

$\Rightarrow$ $r = \frac{4}{2} = 2$ cm

Thus, the radius of the circle is 2 cm.

Q. 15. *Prove that the parallelogram circumscribing a circle is a rhombus.*

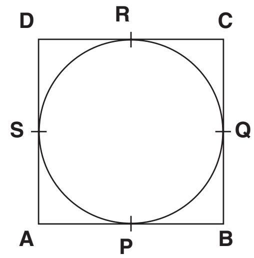
[CBSE 2012] (CBSE Sample Paper 2011)

Sol. Since $ABCD$ is a $\|^{gm}$

$\therefore \quad AB = CD$

and $\quad AD = BC$

$\because$ Tangents from an external point to a circle are equal,



$\therefore \quad AP = AS$

$BP = BQ$

$RC = QC$

$DR = DS$

$\Rightarrow \quad (AP + PB) + (RC + DR) = (AS + DS) + (BQ + QC)$

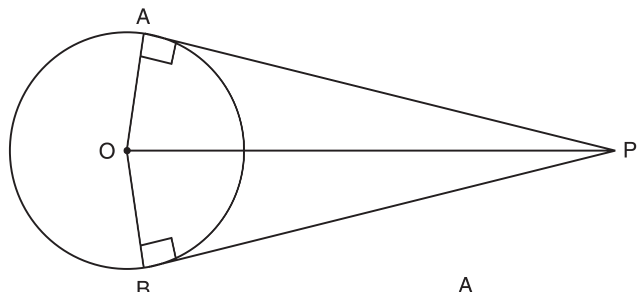
$\Rightarrow \quad AB + CD = AD + BC$

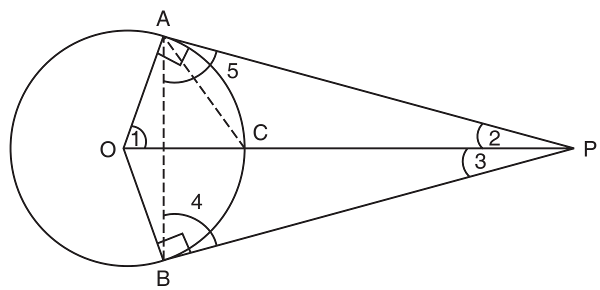
$\Rightarrow \quad 2\,AB = 2\,AD \quad \Rightarrow \quad AB = AD$

$\Rightarrow \quad AB = AD = CD = BC$

i.e., $ABCD$ is a rhombus.

Q. 16. *In the following figure, OP is equal to diameter of the circle. Prove that ABP is an equilateral triangle.* (AI CBSE 2008)





Sol. Since the tangent is perpendicular to the radius through the point of contact,

$\therefore \quad \angle OAP = 90°$

Let us join AB and AC.

In right ΔOAP, OP is the hypotenuse and C is the mid point of OP.

[$\because$ OP is a diameter of the circle (given)]

$\therefore \quad CA = CP = CO =$ Radius of the circle.

$\therefore$ ΔOAC is an equilateral triangle.

Since all angles in an equilateral triangle are 60°,

$\therefore \quad \angle 1 = 60°$

Now, in ΔOAP, we have

$\angle 1 + \angle OAP + \angle 2 = 180°$

$\Rightarrow \quad 60° + 90° + \angle 2 = 180°$

$\Rightarrow \quad \angle 2 = 180° - 90° - 60° = 30°$

Since *PA* and *PB* make equal angles with *OP*,

$\therefore \quad \angle 2 = \angle 3 \quad \Rightarrow \quad \angle 3 = 30°$

$\therefore \quad \angle APB = \angle 2 + \angle 3 = 30° + 30° = 60°$

Again, $\quad PA = PB.$

$\Rightarrow$ In ΔABP, $\quad \angle 4 = \angle 5$ [*Angles opposite to equal sides are equal*]

Now, in ΔABP,

$\angle 4 + \angle 5 + \angle APB = 180°$

$\Rightarrow \angle 4 + \angle 4 + \angle APB = 180°$

$\Rightarrow \quad 2\angle 4 + \angle 60° = 180°$

$\Rightarrow \quad 2\angle 4 = 180° - 60° = 120°$

$\Rightarrow \quad \angle 4 = \frac{120}{2} = 60°$

Since, $\quad \angle 4 = 60°$

$\angle 5 = 60° \quad \because \Delta ABP$ is an equilateral Δ.

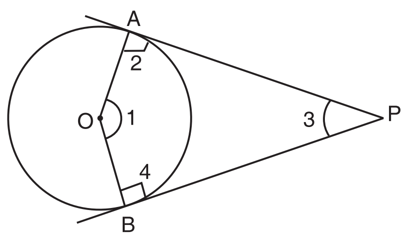
$\angle APB = 60°$

Q. 17. *Prove that the angle between the two tangents to a circle drawn from an external point is supplementary to the angle subtended by the line segment joining the points of contact at the centre.* (CBSE 2008 C)

Or

Two tangents PA and PB are drawn from an external point P to a circle with centre O. Prove that AOBP is a cyclic quadrilateral. (CBSE Sample Paper 2011)

Sol.



We have tangents *PA* and *PB* to the circle from the external point *P*. Since a tangent to a circle is perpendicular to the radius through the point of contact,

$\therefore \quad \angle 2 = 90° \quad \text{and} \quad \angle 4 = 90°$

Now, in quadrilateral *OAPB*,

$\angle 1 + \angle 2 + \angle 3 + \angle 4 = 360°$

$\Rightarrow \quad \angle 1 + 90° + \angle 3 + 90° = 360°$

$\Rightarrow \quad \angle 1 + \angle 3 = 360° - 90° - 90° = 180°$

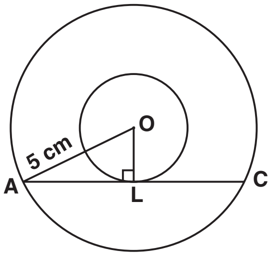
i.e., $\angle 1$ and $\angle 3$ are supplementary angles.

$\Rightarrow$ **$\angle AOB$ and $\angle APB$ are supplementary**

$\Rightarrow$ *AOBP* is a cyclic quadrilateral.

Q. 18. *Out of two concentric circles, the radius of the outer circle is 5 cm and the chord AC of length 8 cm is a tangent to the inner circle. Find the radius of the inner circle.*

[NCERT Exemplar, CBSE (Foreign) 2014]



Sol. Let the given chord AC of the larger circle touch the smaller circle at L.

∵ AC is a tangent at L to the smaller circle with centre O

∴ OL ⊥ AC

Also AC is a chord of the bigger circle

∴ $AL = \frac{1}{2} AC$

[∵ A perpendicular from centre to a chord of the circle, divides the chord into two equal parts.]

But $AC = 8$ cm

∴ $AL = \frac{1}{2}(8 \text{ cm}) = 4$ cm.

Now, in rt. ΔOAL,

$$OL^2 = OA^2 - AL^2$$

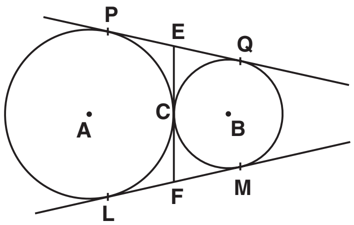
or
$$OL^2 = 5^2 - 4^2$$
$$= (5 + 4)(5 - 4)$$
$$= 9 \times 1 = 9$$

⇒ $OL = \sqrt{9} = 3$ cm

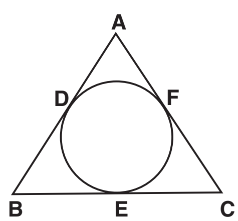
Thus, the radius of the inner circle is 3 cm.

TEST YOUR SKILLS

1. In the following figure, two circles touch each other externally at *C*. Prove that the common tangent at *C* bisects the other two common tangents.



2. In the figure, if *AB* = *AC*, prove that *BE* = *CE*. [CBSE 2006]



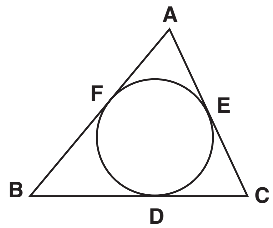
3. A point P is 13 cm from the centre of the circle. The length of the tangent drawn from P to the circle is 12 cm. Find the radius of the circle.

4. A circle is touching the side BC of $\Delta\,ABC$ at P and touching AB and AC produced at Q and R respectively.

Prove that: $$AQ = \frac{1}{2}\text{ (Perimeter of } \Delta ABC)$$

5. The incircle of $\Delta\,ABC$ touches the sides BC, CA and AB at D, E and F respectively. Show that: [CBSE 2012]

$$AF + BD + CD = AE + BF + CE$$
$$= \frac{1}{2}\text{ (Perimeter of } \Delta ABC)$$



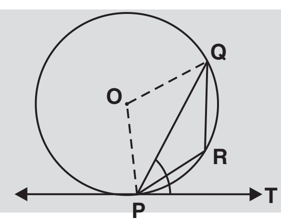
6. Show that the tangents drawn at the end points of a diameter of a circle are parallel.

7. In the figure PQ is a chord of a circle and PT is the tangent at P such that $\angle QPT = 60°$, Find $\angle PRQ$. [NCERT Exemplar]

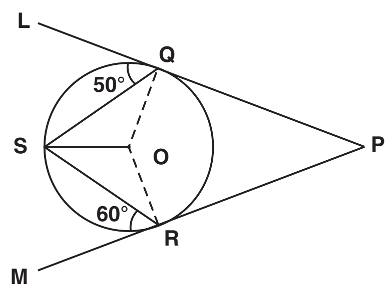
Hint:

$\angle OPQ = \angle OQP = 30° \Rightarrow \angle POQ = 120°$

Also, $\angle PRQ = \frac{1}{2}$ reflex $\angle POQ$



8. In the figure, PQL and PRM are tangents to the circle with centre O at the points Q and R, respectively and S is a point on the circle such that $\angle SQL = 50°$ and $\angle SRM = 60°$. Find the measure of $\angle QSR$. [NCERT Exemplar]



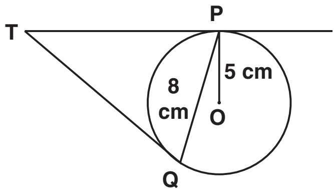
Hint:

$\angle OQS = 90° - 50° = 40° = \angle OQQ$

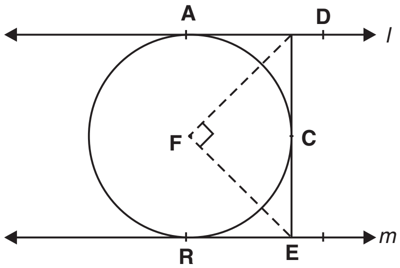
$\angle ORS = 90° - 60° = 30° = \angle OSR$

$\Rightarrow \angle QSR = \angle OSQ + \angle OSR = 40° + 30°$

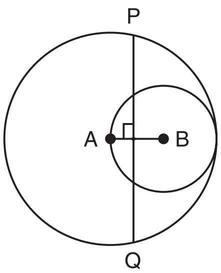
9. PQ is a chord of length 8 cm of a circle of radius 5 cm. The tangents at P and Q intersect at a point T. Find the length of TP.



10. In the following figure, l and m are two parallel tangents to a circle with centre F. DE is the tangent segment between the two parallel tangents touching the circle at C. Show that $\angle DFE = 90°$.

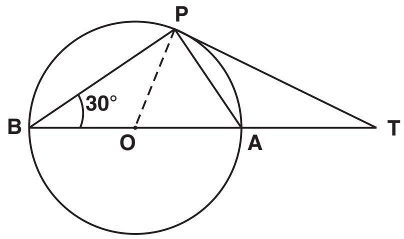


11. In the figure, two circles with centres A and B and radii 5 cm and 3 cm touching each other internally. If the perpendicular bisector of segment AB, meets the bigger circle at P and Q, find the length of PQ.



12. Two tangents making an angle of 120° with each other, are drawn to a circle of radius 6 cm. Show that the length of each tangent is $2\sqrt{3}$ cm.

13. In the figure BOA is a diameter of a circle and the tangent at a point P meets BA extended at T. If $\angle PBO = 30°$, then find $\angle PTA$.



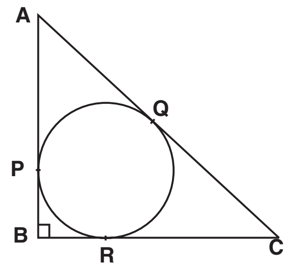
Hint:

$\because \angle BPA = 90°$

$\therefore \angle PAB = 60° = \angle OPA$

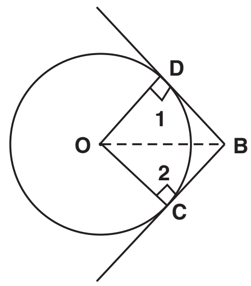
Since, $OP \perp PT \Rightarrow \angle APT = 30°$ *and* $\angle PTA = 60° - 30°$.

14. In the given figure, ABC is a right Δ right angled at B such that BC = 6 cm and AB = 8 cm. Find the radius of the circle.



15. If from an external point B of a circle with centre O, two tangents BC and BD are drawn such that ∠DBC = 120°, prove that BC + BD = BO *i.e.*, BO = 2 BC

[NCERT Exemplar, CBSE 2014]



Hint:

$\angle DOC = 180° - 120 = 60°$ [∵ $\angle 1 = \angle 2 = 90°$]

rt Δ $OBD \cong \Delta OBC$

$\Rightarrow \angle BOC = \angle BOD = 30°$

In rt Δ *OBD*, $\frac{BD}{OB} = \sin 30° = \frac{1}{2}$

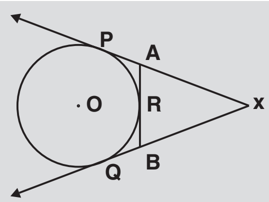
$\therefore BD = \frac{1}{2} OB$

Similarly, $BC = \frac{1}{2} OB$

16. In the figure, XP and XQ are two tangents to the circle with centre O, drawn from an external point X. ARB is another tangent touching the circle at R. Prove that XA + AR = XB + BR.

[AI. CBSE (Foreign) 2014

Hint:



$AP = AR$ *and* $BQ = BR$

XP = XA + AP $\Rightarrow XA + AR$ [∵ $AP = AR$

$XQ = XB + BQ \Rightarrow XB + BR$ [∵ $BQ = BR$

$\Rightarrow$ $XP = XQ$ *gives* $XA + AR = XB + BR$

17. Prove that the line segment joining the points of contact of two parallel tangents of a circle, passes through its centre.

[CBSE (Delhi) 2014]

Hint:

Let PAQ and RBS be two parallel tangents to circle with centre O.

Join OA, OB and draw OC$\parallel$*PQ*

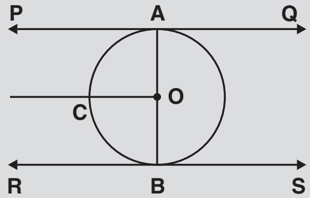
PA||CO

$\Rightarrow \angle PAO + \angle COA = 180°$ [*co-interior angles*]

$\Rightarrow 90° + \angle COA = 180° \Rightarrow \angle COA = 90°$

Similarly, $\angle COB = 90°$

$\angle COA + \angle COB = 90° + 90° = 180°$. *Hence, AOB is a st. line passing through O (centre of the circle).*



18. In the figure, AB is a chord of length 16cm, of a circle of radius 10cm. The tangents at A and B intersect at a point P. Find the length of PA. [AI. CBSE. 2010, 2014]

Hint:

$AB = 16\ cm \Rightarrow AL = BL = 8\ cm$

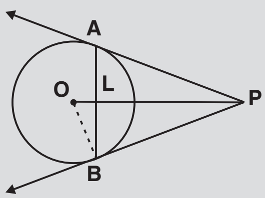
In ΔOLB, $OB^2 = LB^2 + OL^2$ [*By Pythagoras Theorem*]

$\Rightarrow 10^2 = 8^2 + OL^2 \Rightarrow OL = \sqrt{100-64} = \sqrt{36} = 6$

Let $PL = x$ *and* $PB = y$, *So that* $OP = (x + 6)cm$

In ΔPLB *and* ΔOBP, *we have:*

$PB^2 = PL^2 + BL^2$ *and* $OP^2 = OB^2 + PB^2$



Substituting x and y and simplifying, we get $x = \frac{32}{3}\ cm$

$y^2 = x^2 + 64 \Rightarrow y^2 = \frac{1600}{9} \Rightarrow y = 40/3$

Thus, $PA = \frac{40}{3}\ cm.$

ANSWERS

Test Your Skills

3. 5 cm **7.** 120° **8.** 70° **13.** 30° **14.** 2 cm

❑❑❑